* **Q**: [Qishi] What’s the probability that three points on a circle will be on a semi-circle?

**A**:

* **Q**: [Qishi] An ant walks randomly on the edges of a cube. It starts from a vertex, and each step it has equal probability to choose one of the three edges and walk to the other vertex of this edge. What is the expectation of the number of steps for the ant to walk from one vertex to the opposite vertex?

**A**: This problem can be solved by Markov process. Assume walking from vertex A to opposite vertex D. Let B to be the 3 vertices next A and C be the 3 vertices next to D.

Solving the above 3 equations, .

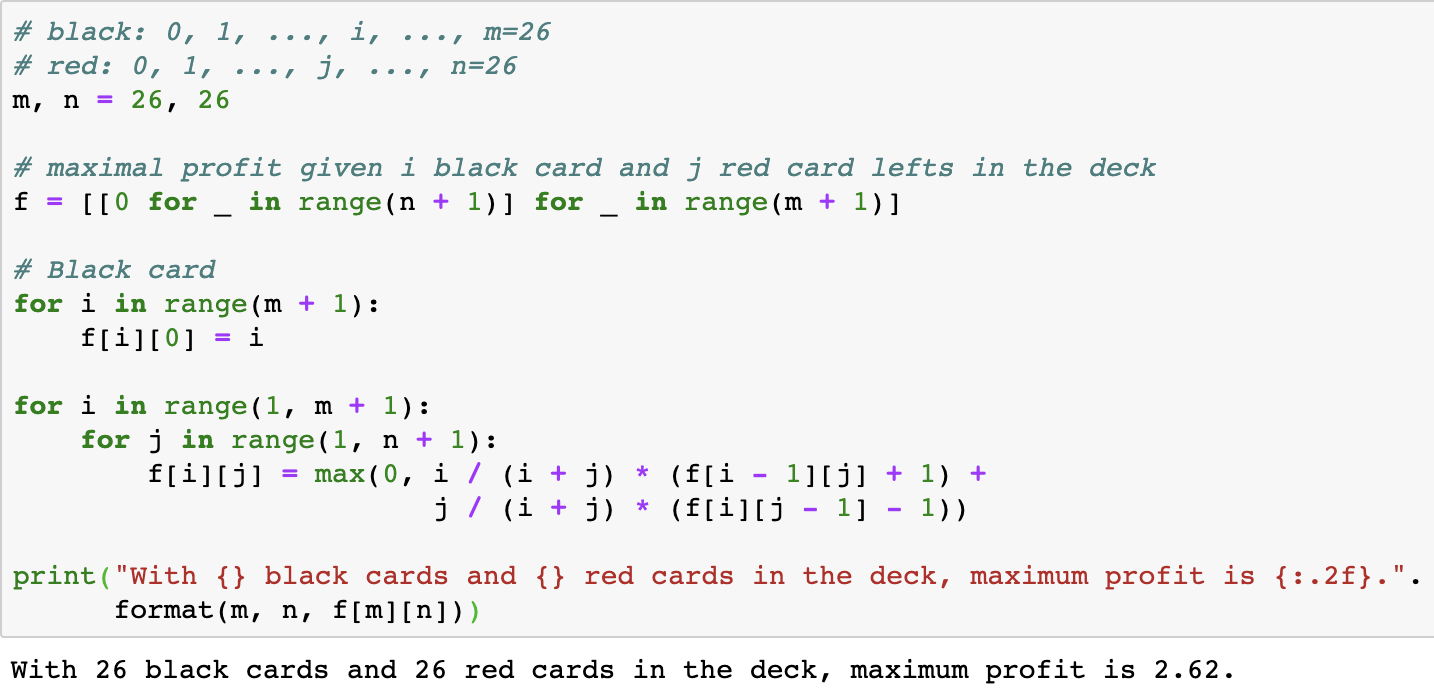
* **Q**: [Qishi] From a deck of 52 cards, you can pick one card each time without replacement. If the card color is black, you win 1$. If the card color is red, you lose 1$. You can stop the game whenever you want. Questions: Will you play the game? If you want, how much would you pay to play this game?

**A**: Define as the expected profit one can get to play the game with black cards and red cards. What we want to calculate is .

Initial conditions:

Dynamic equation:

Python implementation:



* **Q**: [Qishi] Given a coin with probability of landing on heads after a flip, what is the probability that the number of heads will ever equal the number of tails assuming an infinite number of flips?

**A**: This is a random walk problem. Imagine that we are starting our walk at 0, and moving 1 to the right if we get a head, and moving 1 to the left for a tail. We want the probability of returning to 0 sometime after the first flip as this means that we have an equal number of heads and tails.

If the first flip is a head, and we let H = the probability of ever obtaining an equal number of heads and tails starting from 1 head, then H satisfies

That is, we can obtain an equal number on the next flip with probability ( by flipping a tail, or we can flip a head with probability , and then obtain an equal number of heads and tails sometime after that which has probability , since we must make 2 moves to the left, each one of which has probability H, and these are independent. Rearranging and solving for H gives

Now if the first flip is a tail, and we let T = the probability of ever obtaining an equal number of heads and tails starting from 1 tail, then we can proceed as above with replaced by , and replaced by . This results in

Now the overall probability of flipping either a head or a tail on the first flip and then obtaining an equal number of heads and tails is the weighted average of the above two probabilities, or

Therefore, we can conclude that

* **Q:** [Qishi] You have ten light bulbs. Five have an average life of 100 hours, and the other five have an average life of 200 hours. These light bulbs have a memoryless property in that their current age (measured in how long they have already been on) has no bearing on their future life expectancy. Assuming they are all already on, what is the expected number of hours before the first one burns out?

**A**: From the question, we know that the density function of the life of the light bulb follows an exponential distribution.

|  |  |  |  |
| --- | --- | --- | --- |
|  | PDF | E[X] | Var(X) |
| Short-life |  |  |  |
| Long-life |  |  |  |

By the property of exponential distribution, we have

and

So

* **Q**: [Qishi] If a person tosses a coin once per second and he tosses 100 years, whether the following statement is correct or not: the probability of tossing 100 consecutive heads is less than 0.01%.

**A**: Correct. The probability of tossing 100 consecutive head starting from any toss is . By tossing times,

We can calculate the total number of tosses N:

Therefore,

* **Q**: [Qishi] Given a stick, if randomly cut into N pieces, what's the probability that the N pieces can form an N sided polygon?

**A**: We denote the length of piece as . To satisfy the condition that pieces can form an sided polygon, for any piece i, we should have . We can bend the stick and make it a circle, so the question becomes if we can randomly choose N points on the circle such that the probability that these N points are not on a simi-circle. The probability that N points on the semi-circle is . So, the probability that N pieces can form an N sided polygon is 1-.

* **Q**: [Qishi] Suppose in a trading environment, to describe 20 mins prices movement, should we choose moving median or moving average? Why?

**A**: I would say it depends. From robustness perspective, rolling median is better than rolling average, since median numbers are less affected by outliers. From responsiveness perspective, moving average captures the abnormal price change better than rolling median. From computation perspective, rolling median is more computation intense than rolling average.

* **Q**: [Qishi] What is the average number of local maxima of a permutation of 1, …, n, over all permutations? Maxima at ends also count.

**A**: Let’s use the indicator function to denote whether is the local maximum.

Because of the linearity of expectation, we can calculate the average number of local maxima as below.

* **Q**: [Qishi] What do you know about test?

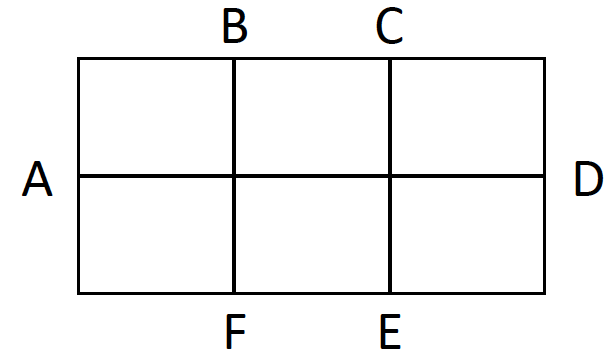
**A**: A chi-squared test, is any statistical hypothesis test where the sampling distribution of the test statistic is a chi-squared distribution when the null hypothesis is true. A chi-squared distribution can be constructed as sum of squared iid normal variables.

It is used to determine whether there is a significant difference between the expected frequencies and the observed frequencies in one or more categories. Assume there are k categories in total, and the probability from category 𝑖 is if null hypothesis is true. We observe 𝑛 samples, and there are observations in category . Then,

with degree of freedom of 𝑘 − 1.

* **Q**: [Qishi] Given a 2 by 3 grid (which has 6 blocks and 17 edges), shortest route to visit all edges (assuming edge length is 1).

**A**: An Euler path in a graph is a walk through the graph which uses every edge exactly once. A graph has a Euler path if and only if there are at most two vertices with odd degrees.



A, B, C, D, E and F are all vertices with odd degrees. In the shortest path, at least 4 points have to be made even points by extra edges, thus, the shortest edges adding to the graph are BC and EF. Now the only two vertices with odd degrees are A and D. The Euler path would be A → D with length of 17 edges + length (BC) + length (EF) = 19.

* **Q**: [Qishi] X, Y are iid , calculate . Try not to use density function of joint distribution.

**A**:

where and .

* **Q**: [Qishi] You have a six-sided dice, you can keep rolling the dice and you get the dollars equal to the amount of the sum. However, if at some point, the sum is a square number, you must stop and will get zero dollars. (1) If at some point, your sum is 35, should you stop or keep rolling? (2) If you choose to continue in the previous question and this is your strategy: you will keep rolling until you reach 43, what is the most probable amount of dollars you would win when you stop? (3) Is there a best strategy for this game, any number that you should stop?

**A**: (1) If the current sum is 35, the next square number is 36 followed by 49. Thus, if we pass 36, we could at least reach 43 before worrying about losing all money again.

You should keep rolling.

(2) The sum value before reaching 43 could be 37 to 42, and we use to to denote the density for each sum value. The stop value could be 43 to 48, and we use to to denote the corresponding density.

(3) There exists at least one state that we need to stop. Otherwise, we will never stop rolling the dice, in which case the expected payoff would be 0 since the probability of hitting some square number is 1.

* **Q**: [Qishi] Given a stick, if randomly cut into 3 pieces, what’s the average size of the smallest, of the middle-size, and of the largest sizes?

**A**: kkk

**Q: [MS] Go down a street, you will see house by house. You have a camera that can only keep one picture. Design an algorithm that can only give equal probability to each house you see along the street (you don’t know the total number of houses when you start).**

A: 1st house: take picture; 2nd house: 1/2 take picture; 3rd house: 1/3 take picture, nth house; 1/n take picture.

**Q: [Citi Quant Trading] There are 6 dimes, 4 nickels, 2 pennies. Randomly choose 6 coins, what’s the probability to get a sum no less than 50 cents?**

A:

**Q: [Citi Quant Trading] The sides of a 10x10x10 cube are painted red and cut into unit cubes how many have at least 1 side of paint.**

A:

**Q: [Citi Quant Trading] Geometric Series The sum of this geometric series to infinity is 3, given a and 1/r are both integers. What are a and r?**

A: a=2, r=1/3

**Q: [Citi Quant Trading] a=16, b=12, c is U[4,28]. What’s probability that a, b, and c can form a triangle?**

A:

**Q: [Citi Quant Trading] a is a prime number, and b is an integer. Find the smallest prime number that satisfies the following equation:**

A: , so must be an integer, so a=13.

**Q: [Citi Quant Trading] You have been given the task of transporting 3,000 apples 1,000 miles from A to B. Your truck can carry 1,000 apples at a time. Every time you travel a mile towards B you must pay a tax of 1 apple but you pay nothing when going in the other direction. What is highest number of apples you can get to B?**

A:

1. make 3 trips with 1000 apples 333 miles. you are at the 667 mi marker and 2001 apples.

2. make 2 trips with 1000 apples 500 miles. you have 1000 apples at the 167 miles. (you have to throw away an apple at the 667 mi marker)

3. 1000 apples and 167 miles = 833 apples.

**Q: [AB] What date (in MMDD:YYYY form) is closest to Jan 1, 2000 and is also a palindrome?**

A: 10022001

**Q: You have three kinds of magazines, all but two are Times, all but two are Science, all but two are Nature. How many magazines in total do you have?**

A: (n-2) \* 3 = n, so n=3

**Q: Blindfolded. In a room with 8 coins, what is the minimum number of flips required to guarantee that one of the permutations had all coins with the same side? Assume that you will know when they are all on the same sides.**

A: 2^(n-1)-1

If n=2, I only need to flip the 1st coin. 1 time

If n=`3, I flip 3rd coin, and 1st coin, and 3rd coin. 3 times

If n=4, I flip 4th, 3rd, 4th, 1st, 4th, 3rd, and 4th coin. 7 times

Insert the new coin (n+1) into the above strategy, and I need to flip extra 2n+1 times.

For n =8, I need to flip 2^(8-1)-1 times.

**Q: Blindfolded. On a squared table, there are 4 coins on the 4 corners. You could flip a coin, a side, or a diagonal. What is the minimum number of flips required to guarantee that one of the permutations had all coins with the same side?**

A: 6 times. Flip order: diagonal – side – diagonal – corner – side - diagonal

**Q: You have 100 balls (50 black balls and 50 white balls) and 2 buckets. How do you divide the balls into the two buckets so as to maximize the probability of selecting a black ball if 1 ball is chosen from 1 of the buckets at random?**

A: You want to put 1 black ball in 1 of the buckets and all of the other 99 balls in the other bucket. This gives you just slightly less than a 75% change of having a black ball chosen.  The math works as follows:  There’s a 50% chance of selecting the bucket containing 1 ball with a 100% chance of selecting a black ball from that bucket. And a 50% chance of selecting the bucket containing 99 balls with a ~49.5% (49/99) chance of selecting a black ball from that bucket. Total probability of selecting a black ball is (50% \* 100%) + (50% \* 49.5%) = 74.5%

**Q: [SIG] 10 friends are attending a party, and each one is going to take 1000 bottles of Pepsi with each one weighs 10 ounces. However, at least one of them is going to cheat, who will bring Cola instead of Pepsi to the party. We know that each bottle of Cola weighs 9 ounces. You also have a digital scale. How you would figure out the cheaters in only one balance?**

A: Take 1, 2, 4, 8, 16, 32, 64, 128, 256, 512 bottles from each person, and weigh the total ounces. The tricky part of this question is that it doesn’t assume there will be only one person who will cheat. Thus, if you take 1, 2, 3, 4.. bottles, you can’t differentiate some situations. For example, if the difference is 3 ounces, then you can’t know whether #3 is cheating or both #1 and #2 are cheating.

**Q: Two blind men are carrying 3 pairs of red rocks and 3 pairs of white socks. They hit each other in the mall and have to redistribute the socks so that they receive the same amount of each color they had before. They can't receive any outside help. How do they do it?**

A: One man took all the socks and pulled the pairs apart. As he pulled them apart, he kept one sock for himself and gave the other to the other man so that each man ended up with the same colors of socks he started with: six red and six white.

**Q: Say we have a pond with lily pads. The lily pads double every minute. After 60 minutes, the pond is completely covered. How long does it take for the pond to be 1/4 covered?**

A: 58min

**Q: Draw circles of any size (but all must be same size thereafter) on a rectangular piece of paper (also any size). The person who can't draw a circle loses.**

**You wanna go first or second? What's your plan?**

A: You want to go first. You draw a circle in the middle of the page. Every time your opponent draws a circle, draw a mirror image. Win.

**Q: [SIG] Two people take turns counting to 50. Each person can add from 1 to 10 to the total sum. The first to count the number 50 wins. What is your strategy? Do you want to go first?**

A: 50 -> 39 -> 28 -> 17 -> 6. I would say 6, 17, 28, 39, and finally 50. Go first.

**Probability**

**Q: [GS] You have a coin with unknown probability of H and T. You flipped it, and found it was an H. What’s the probability that the second flip is also an H?**

A: I would assume uniform distribution of the coin head, which is U[0,1].

**Q: [RBC] Suppose you are given the opportunity to bid for a treasure chest, which you know with 100% confidence to be priced anywhere between $0-$1000. If you bid equal to or above the price, you win the treasure chest (at the cost of your bid). If you bid below the price, you do not earn the treasure chest. Now, also suppose you have a friend who is willing to buy the treasure chest from you for one and a half times the price of the treasure chest (should you obtain the chest). What should your bid be?**

A:

The above equation is always negative, which means you should bid zero.

**Q: What are the odds of a flush in poker if you have 2 hearts in your hand and there are two hearts on the flop?**

A: Taxes Hold’em: flop – run - river

A: 13-4=9; 52-5=47; 47-9=38; 1-Combination(38,2)/Combination(47,2)

**Q: You're playing basketball and you're down by 2. You know you can hit a 2-pointer to tie the game with 28% probability, and then go on to win the game in overtime with 50% probability. Alternatively, you know you can hit a 3-pointer and win with 15% probability. Which do you take??**

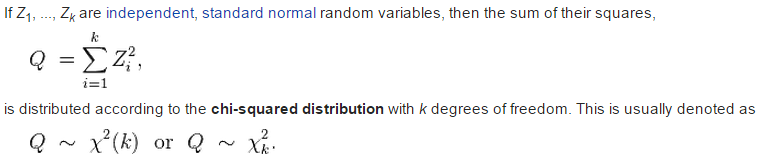
A: 28%\*50%=14% < 15%. Shoot the 3 points.

**Q: We are racing, and can at any time signal to the other that we would like to double our bet. We've put down 100 to start and during the race I signal to you I want to double the bet. What is the minimum probability of winning for you to accept to continue?**

A: Assume the probability to win is p, and 200p + (-200)\*(1-p) = -100. Expected value of continuing must be -100. For this value of p, you are indifferent between betting and dropping out. So for any value of p greater than 1/4, you bet.

**Q: What kind of probability distribution describes the sum of the squares of independent normal random variables?**

A:



**Q: If a submarine can fire and have two torpedoes (missiles) to fire with each one hitting with a probability of 1/3, what is the probability that at least one missile will hit? Chance you hit it twice given that it's sinking.**

A: 1) 1 – (1-1/3)^2 = 5/9

2) P(Two hit | Boat Sinks)  
= P(2 Hit & Boat Sinks) / P(Boat Sinks)  
= 1/9 divided by (5/9)  
= 1/5

**Q: The probability of seeing a shooting star in 1 hour is 91%. What is the probability of seeing a shooting star in 30 minutes?**

A: (1 – p) \* (1 – p) = 1 – 91%, get p = 70%

**Q: What is the probability that three people were born on the same day of the week?**

A: 1/49

**Q: What is the probability of rolling the same side of a dice three times in a row?**

A: 1/36

**Q: I flip n+1 coins and you flip n coins. I win if I flip more heads than you do, and you win if we tie or you flip more heads. What is the probability I win?**

A: 0.5

**Q: What is the probability of being dealt a pair of cards?**

A: For a pair of cards to be dealt from a 52card deck is 3/51 = 1/17. So first card drawn doesn’t matter so you have 3 cards out of the 51 cards left that will match the first hence 3/51 as the probability

**Q: A bunch of people sit down at a circular table. How many combinations are there or certain people sitting down next to others?**

A: It's (N-1)! There are N! ways of situating N people in a line, but since the ends are now connected we have translational invariance, i.e. person 1 could be in any of N spots. Therefore, we are actually over counting by N, so there are N!/N=(N-1)! combinations.

**Q: How many ways can 5 different people sit down at a round table? How many ways can 5 different people sit down in a line of chairs?**

A: 4! 5!

**Q: How many ways can you arrange five people at a round table, such that there they are in increasing age order (clockwise or anti-clockwise)?**

A: 10

**Q: Given 5 people with distinct ages, what is the probability that the five people will sit around a table in ascending order of ages going either clockwise or counterclockwise?**

A: (5\*2) / Permutation(5,5) = 10/5! = 1/12

**Q: 6 people sit at a round table. What is a probability that they are in height order?**

A: 6 \* 2 / 6! = 1/60

**Q: (1) I roll a dice and pay you the amount showing (so if I roll a 4, I give you $4). (2) Same question, except this time you are given the choice to re-roll after the initial roll, making the initial result null you are paid based on the second roll. How much would you pay to play this game? (3) What about rolling three times?**

A: 1) 3.5 The first roll is worth $3.5.

2) 2 \* 1/4 + 5 \* 3/4 = 17/4 = $4.25 If you get to roll twice, you need some game theory. You wouldn't take roll 1 if it wasn't 4, 5, or 6. So the average of what you would take is 5. You will get these numbers 1/2 of the time (4,5,6 make up half the numbers on the die)  
0.5\*5 + 0.5\*3.5 = 2.5 + 1.75 = 4.25  
The other 0.5 of the time you get the average roll of a die(.5\*3.5)  
3) For the third roll(it will be asked sometimes), you know 4.25 is the value of 2 rolls. So for the first roll, you only keep 5 and 6, or 5.5 averaged. You get these 1/3 of the time.  
1/3\*5.5 + 2/3\*4.25 = 1.8333 + 2.833 = 4.667

**Bayes Formula**

**Q: [GS] 10% of people has a disease. If the person has the disease, the device has 100% chance to detect it. If the person doesn’t have the disease, the device has 10% chance to detect it wrong. What’s the probability that the person has the disease given that the device says that the person has it?**

A:

**Q: [SIG] There are three kinds of smokers: heavy-smokers with prob. 20%, light-smokers with prob. 30%, and non-smokers with prob. 50%. The prob. of death of light-smokers is the twice of the prob. of non-smokers, and the prob. of death of heavy-smokers is the twice of the light-smokers. Question is what’s the probability of the death of a person is a heavy-smoker?**

A: Assume the prob. of the death of a non-smoker is x, then the prob. of light-smoker is 2x, and the prob. of the heavy-smoker is 4x.

**Q: [SIG] A bag contains 5 coins. Four of them are fair and one has heads on both sides. You randomly pulled one coin from the bag and tossed it 5 times, heads turned up all five times. What is the probability that you toss next time, heads turns up. (All this time you don't know you were tossing a fair coin or not)**

A:   
P(6th is H/5H in a row) = P(6H in a row)/P(5H in a row)

P(5H in a row) = P(5H/unfair)P(unfair) + P(5H/fair)P(fair) = (1/2)^5 \* 4/5 + 1/5 = 9/40   
P(6H in a row) = P(6H/unfair)P(unfair) + P(6H/fair)P(fair) = (1/2)^6 \* 4/5 + 1/5 = 17/80   
so correct answer is (17/80)/(9/40) = 17/18

**Q: [SIG] We have 10 coins. 9 of them are fair, which have heads and tails. 1 of them has double heads. You have flipped them 5 times, and you got 5 heads. What’s the prob. that you get another head for the 6th flip?**

A:

**Q: [SIG] You know there are 3 boys and an unknown number of girls in a nursery at a hospital. Then a woman gives birth to a baby, but you do not know its gender, and it is placed in the nursery. Then a nurse comes in picking up a baby and it is a boy. Given that the nurse picks up a boy, what is the probability that the woman gave birth to a boy?**

A: So I've narrowed it down (taking the number of girls to be *n*) to:

*P*(woman had a boy∣nurse picked up boy)

*=P*(nurse picked up boy∣woman had a boy)*P*(woman had boy) / *P*(nurse picked up boy)

= [1/2 \* 4/(n+4)] / [1/2 \* 4/(n+4) + 1/2 \* 3/(n+3)]

= 4/7.

**Q: [SIG] I want to have a birthday party outside this weekend (i.e. need sunny weather). On Saturday, the chance of rain is 60 percent, sun 40 percent. On Sunday, rain is 80 percent, sun is 20 percent. From there, the interviewer can ask an array of questions: (1) What are the chances I can have my party this weekend? (2) If I have my party, what are the chances my party is on Saturday/Sunday? (3) What is the probability that that it rained at the weekend? (4) What’s the probability that it rains on the weekend given that you had the party?**

A: 1. P(party)=1-P(no parties)=1-(0.6)\*(0.8)=0.52  
2. P(Sa | party)= P(Sa & party)/P(party) = 0.4/0.52 = 10/13 (since, sun on Saturday guarantees party on that day, that's how we get the numerator)  
3. P(Su | party) = P(Su & party)/P(party) = P(rain Sa and sun on Sun)/P(party)  
                            = (0.6)(0.2)/0.52 = .12/.52 = 3/13  
OR: just use P(Su | party) + P(Sa | party) =1

4. P(rained on weekend) = 1-0.4\*0.2=0.92

5. A simple way of doing this is to look at the sample space  
RR - rain both days = 0.8 \* 0.6 = .48  
RS - rain Saturday sun Sunday = 0.8 x 0.4 = 0.32  
SR - sun Saturday rain Sunday = 0.2 x 0.6 = 0.12  
SS - sun both days = 0.2 x 0.4 = 0.08

Since you had the party RR is out of the picture,   
P(Rain over the weekend I You had the party)

= P(Rain over the weekend & had the party)/P(had the party)

=(RS+SR)/(SS+RS+SR) = (.32+.12)/(.08+.32+.12) = 44/52 = 11/13

**Expectation**

**Q: [GS] (1) How would you generate a uniform distributed 1, 2, 3, 4, 5, and 6 using a fair coin? (2) How would you generate a uniform distribution U[0,1] using a fair coin? (3) How would you generate a uniform distribution U[0,1] using an unfair coin?**

A: (1) Flip the coin 3 times, and we will get an event set {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT} with each event 1/8 probability. Whenever we got the last two events, we ignore them and flip the coin again. (2) I treat H as 1, and T as 0. Assume N is the total number of flips, then

(3) For an unfair coin,

**Q: [World Quant] You have one unfair coin with 80% to get a H, and 20% to get a T. How would you get equal probability using this coin? What’s the expected number of flips to get the event?**

A: Always toss twice. Prob(HT)=Prob(TH)=0.16, and ignore the HH and TT events. Let X be the number of fairs needed to obtain the events set {HT, TH}. It’s a Geometric probability distribution.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Pmf | E[X] | Var[X] |
| Geometric |  |  |  |

Thus,

E[Number of Flips] = E[X] \* 2 = 1/0.32\*2 = **6.25**

**Q: [GS] What’s the expected number of tosses of a dice to get all the sides of it?**

A: Geometric distribution:

**Q: [GS] You have three balls with numbers 1, 5 and 10. You randomly pick up one ball, and write down the number, put it back to the pool, and randomly pick up one ball again. You summarize all the numbers until the sum is larger than or equal to 9. What the expected value of the sum?**

A: Define , then

**Q: [SIG] You have a deck of 97 cards and I will pay you $10 if I draw 4 cards and they are in ascending order (not necessarily consecutive order) and you pay me $1 if they are not. Would you play?**

A: 97 is the red herring. Once you have drawn four cards without replacement, you know that they are all of different values. There are 24 different ways that they can be ordered, and by symmetry they all are equally likely.

EV = 10 \* 1/24 – 1 \* (1-1/24) = -13/24. Don’t play.

**Q: [SIG] I have 10 cards face-down numbered 1 through 10. We play a game in which you choose a card and I give you the corresponding dollar amount.  
a) What is the fair price of this game?  
b) Now, after picking a card you can either take the dollar value on the card or $3.50. Also, cards worth less than 5 are now valued at $0. What is the maximum price you are now willing to pay for the game?**

A: a) EV = ((1+10) \* 10) / 2 / 10 = $5.5

b) EV = 0.4 \* 3.5 + 0.6 \* 7.5 = $5.9

**Calculus & Linear Algebra**

**Q: [MS] y=f(x), f(f(x)) = x+1. Is it possible to have a constant function y=f(x)=c to satisfy the above equation?**

A: No. f(f(f(x))) = f(x+1) = f(x) + 1. If there exist such an constant that y=f(x)=c, then c=c+1, which doesn’t make sense.

**Q: [MS] Given you have three normal random variables with the correlations with each other is the same. How would you calculate the range of the correlation?**

A: Correlation matrix must be positive semidefinite. The correlation matrix:

The determinate must be larger or equal to zero.

Solve the above equation, the range of the correlation is [-0.5, 1.0].

**Q: A ball w/ radius 0.5m, the radius is changing at 4m/s. Calculate the Volume change of this ball w.r.t. the change of time.**

A:

**Q: Known E[X], E[Y], Var[X], Var[Y], corr(x,y). Calculate Var(aX+bY).**

A:

**Q: If I drop a ball from 10 meters high, it will always rebound half the distance, e.g. the first time, it rebounds 5m high, then the second time 2.5m, etc. So how long does the ball travel after it finally stops?**

A:

Total distance of travel = 10 + 5 + 5 + 2.5 + 2.5 + …

= 10 + 2 \* 5 \* (1 + 1/2 + (1/2)^2 + … )

= 10 + 2 \* (5/(1 – 1/2))

= 10 + 2 \* 10

= 30

**Finance**

**Q: What’s put-call parity?**

A: Options with same K and T:

European Option with dividend:

American Option without dividend:

**Q: What’s implied volatility? What’s the difference between historical volatility?**

A: Implied volatility of an [option](http://en.wikipedia.org/wiki/Option_(finance)) is that value of the [volatility](http://en.wikipedia.org/wiki/Volatility_(finance)) of the underlying instrument which, when input in an [option pricing model](http://en.wikipedia.org/wiki/Valuation_of_options) (such as [Black–Scholes](http://en.wikipedia.org/wiki/Black%E2%80%93Scholes_model)) will return the current market price of the option. Implied volatility, a forward-looking and subjective measure, differs from historical volatility because the latter is calculated from known past returns of a [security](http://en.wikipedia.org/wiki/Security_(finance)).

**Q: What is beta? What is correlation?**

A:

**Q: [Citadel] You have one stock trading at $1000, 1/3 chance of going up to $1090, and 2/3 chance of going down to $940. You have one call option with strike $1030 with premium $30. Assume interest rate is 0, how would you arbitrage this opportunity?**

A:

* Method 1: Replicating portfolio

Assume you have x shares of stock, and y value of bond, then you have 1000x+y value at time t0. At time t1, the portfolio has two states:

It means that you need to long 0.4 share of stock and short $376 value of bond to replicate the portfolio. By no-arbitrage condition, if you have two portfolios with the same payoff at time t1, then you should have the same price at time t0. So, the fair price of the call option should be 1000x+y=4000-376=**$24**.

You could short the call option with premium $30, and long the replicating portfolio to hedge it.

* Method 2: Risk Neutral Pricing

1/3 chance of going up and 2/3 chance of going down are both real probabilities. You should use risk-neutral probability to price an option.

Then the fair price of the option = 60 \* 0.4 + 0 \* 0.6 = **$24**

**Q: How would you hedge a forward contract long position?**

A: (1) Short a call and long a put to hedge the position.

Long a forward is equal to long a call and short a put of the same underlying security with the same strike price and same expiry date. Put-call parity:

where C is the current value of a call, P is the current value of a put, D is the discount factor, F is the forward price of the contract, and K is the strike price.

(2) Short stock and long bond.

Long forward is also equal to long stock and short bond (borrow money).

**Q: How do hedge $1million stock long position?**

A: There are three ways to hedge stock position.

(1) *Options hedge*: more reliable and provides leverage, but requires a premium. If you are long a stock and you want to reduce downside risk, you will want to buy put options. If you are short a stock, and you want to reduce upside risk, you should buy call options.

You will have to find the options appropriate for your timeframe, the strike price you are willing to take, and the premium you are willing to pay. You will want to pick a strike price you are happy with - if you have a strict discipline of cutting all your losses to a maximum of 10%, then you can choose an option with a strike price of up to 10% away from your purchase (10% below with a put option if you are in a long position, 10% above with call option if you are short). Keep in mind there is nothing that says the strike has to be 10% away, it can be 5% away, 2% away, even the purchase price, as long as it fits your comfort level. If you are already profitable, you can keep a put option above your purchase price or call option below the short price to lock in some of the gains. Also keep in mind that the premium of an option is partly dictated by its strike price.

|  |  |
| --- | --- |
| Options | Stocks |
| Long/Short 1 call | Short/Long *Delta* shares |
| Short/Long ***1/Delta*** call | Long/Short **1** share |
| Long/Short 1 put | Long/Short Delta shares |
| Long/Short ***1/|Delta|*** put | Long/Short 1 share |

(2) *Pairs hedge*: not reliable. If I have a position of stock A, I can hedge it by holding the opposite position of its competitor in the same industry with total values of the two positions equally. This is to protect the industry risk, as most of the stocks of one sector will follow each other in large market movements.

Assume you found company A’s competitor company B in the industry. So the question becomes how many shares of company B stock should you short to minimize the variance of the hedged position. The variance of the portfolio return:

Take the first derivative of the portfolio variance with respect to n:

Thus, for every unit value of stock A long position, I should short n value of stock B position. Thus, totally, I should short **$1\*n million** value stock B to hedge the long position of stock A.

(3) *Futures hedge*: more risker because of highly leveraging. If you are long a stock and you want to reduce downside risk, you will want to short contracts. If you are short a stock, and you want to reduce upside risk, you should buy contracts.

Similar to options, you will want to watch for the strike price, premium and time frame. The difference with contracts is that you are obligated to fulfill the contract at the time of expiry. You should find the appropriate strike price at the appropriate time.

Suppose you are a supplier wanting to sell a shipment of silver for at least $14.00 in July. The price of silver is $14.00 now (March), but you cannot sell now because your shipment is not in yet, but you are worried of a price drop. This is when you would write a July Silver contract for $14.00.

Come July, and if the price is $12.00, you will have locked in your sale at $14.00, and have saved yourself $2.00 on each ounce. However, if the price rises to $16.00, you will still have to fulfill your contract, and sell your silver at $14.00, missing out $2.00 for each ounce. With futures, you lock in your potential return at that certain price to be fulfilled at that certain time.

**Q: If you short 1million AAPL stock, and you know that the beta of AAPL is 1.1, how would hedge the systematic risk using SPY?**

A: Long 1million\*1.1 SPY.

**Q: Do you think gold is a hedge against inflation?**

A: No. I think gold is a very speculative product, and it’s a hedge of nothing in normal economic situation. But gold can a hedge against crisis. When the global economy is really bad and investors are afraid to buy equities, bonds, or currencies, they might prefer holding gold.

**Q: What’s the TED spread?**

A: TED spread is now calculated as the difference between the three-month LIBOR and the three-month T-bill interest rate. The size of the spread is usually denominated in [basis points](http://en.wikipedia.org/wiki/Basis_point) (bps). For example, if the T-bill rate is 5.10% and ED (Eurodollar Futures Contract) is at 5.50%, the TED spread is 40 bps. A rising TED spread often indicates a downturn in the U.S. stock market, as it indicates that [liquidity](http://en.wikipedia.org/wiki/Liquidity) is being withdrawn.

**Stochastic Calculus Questions**

**Q: Write down Geometric Brownian Motion. What the graph look like if sigma is zero? Do you believe that the graph is correct for US stocks?**

A:

Solve the above SDE,

When =0, the graph is exponential:

During 1990 to 2010, the graph is pretty accurate.

**Q: What is Black-Scholes model? What are the assumptions behind it?**

A:

*Assumptions:*

1. No arbitrage in the market.

2. No market friction: no transaction cost, no tax.

3. There is a constant risk-free rate.

4. There is no limit of cash borrowing and lending at the risk-free rate.

5. The stock price is lognormal distributed.

6. The stock doesn’t pay dividend.

**Q: What’s fundamental theorem of asset pricing?**

A: There are two fundamental theorem of asset pricing. The first fundamental theorem of asset pricing model is that the market is no-arbitrage if and only if there exists at least one risk-neutral probability measure that is equivalent to the original probability measure. The second fundamental theorem of asset pricing is that the market is complete if and only if there exists a unique risk-neutral probability measure that is equivalent to the original probability measure.

**Q: Why use risk-neutral probability measure, not real probability measure?**

A: According to fundamental theorem of asset pricing, in a complete market, a derivative’s price is the present value of the expected future payoff under the unique risk-neutral probability measure.

If using the real probability to price assets, the expected values need to be adjusted for an investor’s risk preference. But the risk preferences are different among investors, and also difficult to quantify. Risk-neutral pricing doesn’t need to consider the investors’ risk preference.

**Q: What is martingale?**

A: Martingale is a [sequence](http://en.wikipedia.org/wiki/Sequence) of [random variables](http://en.wikipedia.org/wiki/Random_variable) that, at any particular time, the [expectation](http://en.wikipedia.org/wiki/Expected_value) of the next value is equal to the present value even given all the prior [observed values](http://en.wikipedia.org/wiki/Realization_(probability)).

**Q: What is Ito formula?**

A: Ito formula is used to calculate the differential of a [stochastic process](http://en.wikipedia.org/wiki/Stochastic_process) w.r.t time and a Brownian motion. The stochastic process has the form of

Then any twice differentiable function w.r.t. has the differential form: